Professional Goals

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My scholarship is focused on two primary goals: advancing research in general topology, and the enhancement of cyberinfrastructure necessary for cutting-edge advancements of mathematics research and STEM higher education. I am also very active in mathematics outreach, particularly through my efforts as a puzzle and game designer.

Research

My research in general topology focuses in particular on topological properties that can be characterized in terms of infinite combinatorial games. I often use the following toy example for communicating this work to general audiences. When playing Tic-Tac-Toe on a full sheet of paper, both players have a strategy that will prevent their opponent from winning, so a “perfectly played” game always results in a tie. But if two opposite edges are taped or glued together, this changes the topological structure of the space the game is played on, as tic-tac-toes can now loop around where the edges were identified. In this case, the first player now has a strategy that always guarantees victory in the game, regardless of how well the second player chooses their moves. Thus, the presence of a winning strategy for the first player in Tic-Tac-Toe may be thought of as a consequence of the topological structure of the paper it’s played on.

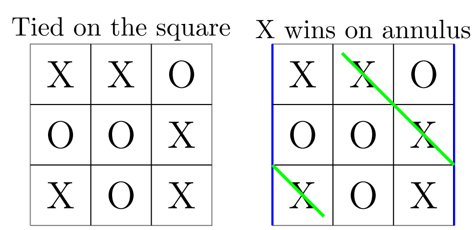

Illustration of the relationship between topological structure and game theory.

Of course, the games and spaces studied in my research are more elaborate. A topological game generally is played over infinitely-many rounds, with each player alternating making a choice based upon the selections made in previous rounds. Such games cannot be completed, but players may still define a strategy, that is, a mathematical function that defines the choice they should make in response to every possible finite combination of moves an opponent might make, and then by comparing chosen strategies for both players, one may calculate which strategy results in a win for its user. By defining games appropriately, we may then characterize a property of a space (analogous to whether the Tic-Tac-Toe paper is flat or looped) based upon which player (if either) has a “winning” strategy that defeats all possible counter-strategies by the opponent.

One such game is known as Menger’s game. For a given topological space, during round N player ONE chooses an open cover U_N of the space, and player TWO responds by choosing some finite subcollection F_N of U_N. If the union of TWO’s choices F_N for each natural number M itself forms an open cover of the space, then TWO has won the game, and ONE wins the game otherwise. The game is named for Menger due to this classic result of Hurewicz: ONE lacks a winning strategy in this game if and only if the space under consideration has the so-called Menger covering property. One of the results from my earliest work established a dual result: for a regular topological space, TWO has a winning Markov strategy in this game (relying on only the most recent move of the opponent and the current round number) if and only if the space is a countable union of compact subspaces.

Menger's game is a specific example of the generalized selection games $G_1(A,B)$ and $G_{fin}(A,B)$. In these games, ONE chooses a member $a_N \in A$ each round, and TWO chooses a singleton (for G_1) or finite (for G_{fin}) subset b_N from this choice. If the union of TWO's choices $\bigcup_{N<\omega} b_N$ belongs to B, then TWO has won the game, and ONE wins the game otherwise. By letting A and B equal various topological collections (various kinds of open covers, convergent sequences, dense subsets, and so on), the game $G_{fin}(A,B)$ can be used to characterize relevant properties related to those collections. As such, much of my research has focused on analyzing the selection games $G_1(A,B)$ and $G_{fin}(A,B)$ for generalized sets A, B, such as the following selected results.

Theorem 1 ([4]) *If the union of A is countable, and any superset of a member of B also belongs to B, then TWO has a winning strategy for $G_{fin}(A,B)$ if and only if TWO has a winning Markov strategy for $G_{fin}(A,B)$.*

Theorem 2 ([5]) *If the set A has a "reflection" R (for example, the collection of open covers of a space's reflection is the collection of local bases of the space), then the games $G_1(A,B)$ and $G_1(R,\neg B)$ are dual. That is, ONE has a winning strategy in either game if and only if TWO has a winning strategy in the other game.*

Theorem 3 ([6]) *If A is "Γ-like" (for example, the collection of sequences converging to a point) and contains B, then ONE has a winning strategy in $G_{fin}(A,B)$ if and only if ONE has a winning strategy that ignores the moves of TWO.*

In addition to this line of research, I have also contributed to continuum theory and the study of topological inverse limits. In particular, most inverse limits are indexed by simple linear orders such as the natural numbers or integers. However, my work generalizes such characterizations to allow for any linearly ordered index, and working with a Master's student we published several results from his thesis that allow generalizing indices even further to partial orders and preorders. In particular, no matter how the index is ordered, the inverse limit will be metrizable if and only if the index is countable (modulo trivial exceptions). As of writing, I'm currently working on extending this result to determine when generalized inverse limits are continua; in particular, extending known results for standard inverse limits that characterize when connectedness is preserved.

Cyberinfrastructure

While a graduate student, I partnered with a colleague to found a start-up company, offering Customer Relationship Management services for academic music organizations through a custom web application we developed together. We dissolved this company several years ago when I accepted my first academic job, but I've continued to use my skills as a business manager and software engineer to contribute to the cyberinfrastructure of mathematics research and STEM higher education.

In 2018 I was invited by the American Institute of Mathematics (AIM) to join the community of practice surrounding PreTeXt [1]. PreTeXt is a language used for authoring scholarly documents in STEM such as textbooks and research articles; the ActiveCalculus.org textbook series is widely considered the flagship product utilizing PreTeXt. In particular, a single PreTeXt-authored source file can produce LaTeX/PDF output, accessible HTML with interactive elements, tactile Braille pages, and more. My major contribution to PreTeXt software has been the co-development of the PreTeXt-CLI package, streamlining the installation and use of PreTeXt.

Additionally, I lead the development of two other software tools used to support higher education in STEM. The CheckIt application [2] provides scaffolding to author randomized mathematics exercises using the open-source mathematics language SageMath. CheckIt-generated exercises can then be exported to HTML, LaTeX, or PreTeXt, and also used for randomized quizzes in the Canvas LMS. Additionally, my Scratchee [3] app provides a free mechanism for utilizing the Immediate Feedback Assessment Technique (IF-AT) [8]. Students working in groups answer multiple-choice questions in the app; when an incorrect choice is selected, they are told immediately, and after reflection they may select another choice for partial credit.

My work on these software products is related to several National Science Foundation grants. PreTeXt development has been supported by several NSF awards, most recently #1821706 funding $1.1 million to develop PreTeXt and study the efficacy of open-source textbooks in mathematics higher education, for which I served as a consultant. I am directly an investigator on two active NSF projects related to this work as well. My team was awarded $579,762 for our Transforming Lower Division Undergraduate Mathematics Through Team-Based Inquiry Learning project, #2011807; my main contribution to this project has been leading the development of PreTeXt and CheckIt open educational resources for instructors that wish to adopt Team-Based Inquiry Learning. The Scratchee app was also developed as part of this work, eliminating the requirement of commercial solutions for IF-AT assessment.

Moreover, I serve as principal investigator of our recently-funded Phase I POSE award: An Open-Source Ecosystem for the Creation and Use of Accessible Science, Technology, Engineering and Mathematics (STEM) Open Education Resources, #2230153. This one-year, $267,268 award is focused on scoping the development of a formal Open-Source Ecosystem surrounding PreTeXt and related software products. POSE is a new solicitation, part of NSF's newest Directorate for Technology, Innovation and Partnerships (TIP), established in 2022. Our award is one of twenty-four in the inaugural round of POSE funding; it's expected that roughly ten of these projects will move on to receive $1.5 million over two years in Phase II funding. In particular, the establishment of the TIP directorate and solicitations such as POSE reflect NSF's increasing support for the development of technologies and cyberinfrastructure that will enable next-generation advancements in STEM and STEM education.

I am currently active in one particular next-generation advancement in mathematics research, serving as mathematical editor of the π-Base Database of Topological Spaces [7]. This service catalogues topological spaces, properties, and theorems connecting these properties. For example, contributors may tag a given space as compact and Hausdorff, and the π-Base's automated deduction uses the theorem `compact+Hausdorff=>normal` to immediately tag the space as normal. Future work on the π-Base will improve the user experience to allow contributions via a web browser rather than local installation, and integrate π-Base's content with the `mathlib` library of formalized mathematics expressed in Lean.

In addition to continuing development of the above products, my future plans for enhancing the cyberinfrastructure of mathematics research include the development of a replacement for the deprecated Topology Atlas [9]. Shuttered in 2020, Topology Atlas was designed to be a comprehensive online resource for topology researchers, and provided services such as Q&A forums and abstract submission services for conferences. While some services such as Math.StackExchange fill some of the roles once served by the Atlas, several conference organizing committees that once relied on the Atlas now have been left to reinvent the wheel. Based on my discussions with organizers of the Spring Topology and Dynamics and Summer Topology Conference, I believe this is a significant need that extends beyond the scope of topology research.

Outreach

During graduate school, I became very active in playing and designing puzzlehunt events, akin to the Microsoft Puzzle Challenge and MIT Mystery Hunt. In these games, players solve hidden-message puzzles that send them to locations throughout a city or campus. In recent years this kind of game has been consolidated and popularized for general audiences as "escape rooms" across the world.

Working with another graduate student, we adapted the puzzlehunt format to create the Auburn Mathematical Puzzle (AMP'd) Challenge, and after completing our PhDs we founded the Mathematical Puzzle Programs (MaPP) organization to create and run similar games at colleges across the country. Puzzles in these games frequently are based on ideas from areas of mathematics not seen in the usual secondary math classroom, such as graph theory or topology. Working with a math education researcher at Southern Connecticut State, we have begun a research project to measure how participation in such games improves students' mathematical identities, particularly for students underrepresented in mathematics. After this preliminary work is complete, we will seek external funding to continue this investigation.

My work involving mathematical puzzles and games has led to many useful collaborations with the National Museum of Mathematics, the Julia Robinson Mathematics Festival, MathCommunities.org, and more. I have authored two books of logic puzzles for general audiences myself, and was invited by the AMS Notices to review the book *Games for Your Mind* on the connections between logic puzzles and foundational mathematics. I plan to continue writing books and designing games that engage general audiences in mathematical thinking, eliminate mathematical anxiety, and improve mathematical identities.

References

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